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On the Calculation of Wake Functions Using Mafia-T3 Code*

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1 Wake Functions

A test charge at (r, θ, z) trailing a point source at (a, θ_0, z_o) experiences a wake force. This wake force can have both a longitudinal component and a transverse one. From Refs. [1] and [2] , the general form of the wake force components are given as:

$$F_z(s) = -\sum_{m=0}^{\infty} eI_m W'_m(s) r^m \cos m(\theta - \theta_0) \ , \tag{1}$$

$$F_\perp(s) = \sum_{m=1}^{\infty} eI_m W_m(s) m r^{m-1} [\hat{r} \cos m(\theta - \theta_0) - \hat{\theta} \sin m(\theta - \theta_0)] + F_\perp^0(s) \ , \tag{2}$$

where e is the charge of the test particle, $s = z_o - z$ is the longitudinal distance the test particle is lagging behind. The multipole coefficients of the point source are $I_m = qa^m$, where q is the charge of the source. Here, the wake force components have been integrated across the structure of the vacuum chamber; i.e., $F_z(s)$ and $F_\perp(s)$ have dimension [force*length]. The function $W_m(s)$ is called the transverse wake function or wake potential in the m-multipole and W'_m is the corresponding longitudinal wake function. The axis of the cylindrical coordinate is chosen as the path of be particle beam, along which coupling impedances are to be evaluated. With respect to this axis the structure being studied may not have cylindrical symmetry. Under this situation, strictly speaking Eqs. (1) and (2) are not valid. However, we expect Eqs. (1) and (2) to hold when the offsets r and a of the test particle and source particle are sufficiently small, and this is actually what we need in the computation of impedances. For structures with no axial symmetry, the $m = 0$ component represented by $F_\perp^0(s)$ in Eq. (2) is in general nonzero.

In the case of cylindrical symmetry, TBCI computes each multipole m of the wake functions separately by setting $r = a =$ pipe radius to eliminate numerical noise. The transverse wake potential $W_m(s)$ is obtained by integrating $W'_m(s)$.

The 3-D MAFIA-T3 code [3], without any assumption of cylindrical symmetry, computes the total wake force separately for both the transverse and longitudinal components. It is clear from Eq. (1) that the lowest harmonic of the longitudinal wake function, $W_0(s)$, can be computed without offsetting the beam. For the transverse wake function, however, one must offset the beam by a. In accordance with TBCI, the transverse forces are computed at $r = a$ and $\theta = \theta_0$. The form of Eqs (1) and (2) become:

$$F_z(s) = -\sum_{m=0}^{\infty} eqW'_m(s)a^{2m} \ , \tag{3}$$

$$F_\perp(s) = \sum_{m=1}^{\infty} eqW_m(s)ma^{2m-1} + F_\perp^0(s) \ . \tag{4}$$

The dipole transverse wake is therefore given by:

$$W_1(s) \approx \frac{F_\perp(s) - F_\perp^0(s)}{eqa} \ , \tag{5}$$

provided that the higher multipoles do not contribute appreciably when a small a is chosen.

Since TBCI is so much different from MAFIA-T3, a comparison has to be made. For this purpose we select a cylindrically symmetrical smooth pillbox cavity. Results from the two codes for both the monopole and dipole of the longitudinal wake potentials and the dipole component of the transverse wake potential are compared. Similar calculations are made for an asymmetrical cavity using MAFIA-T3 only.

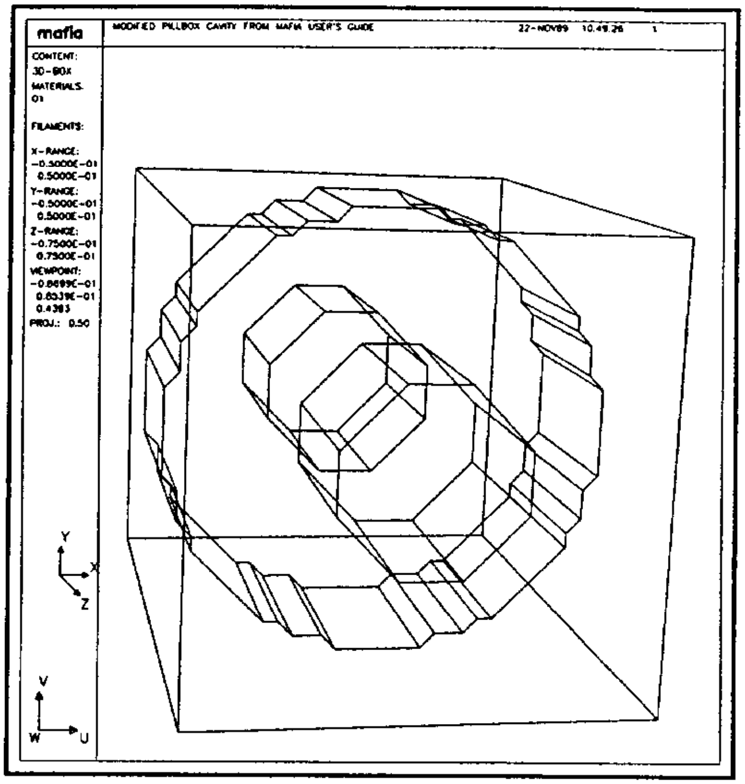

Figure 1: Problem geometry used in TBCI and MAFIA

2 The Symmetric Pillbox Cavity

The problem geometry is shown in Fig. 1. It consists of a cylindrical cavity of radius 5 cm and a length equal to 5 cm. The beam pipe is of radius 2 cm extruding 5 cm at each side. This geometry is modeled using both MAFIA and TBCI. In both cases, the source bunch is a gaussian truncated at $\pm 5\sigma$, where $\sigma = 5$ cm is the standard deviation.

2.1 Longitudinal Wakes

The longitudinal wake as obtained by MAFIA T3, for three different mesh sizes and offsets, are given in Fig. 2. This graph shows little dependence of the

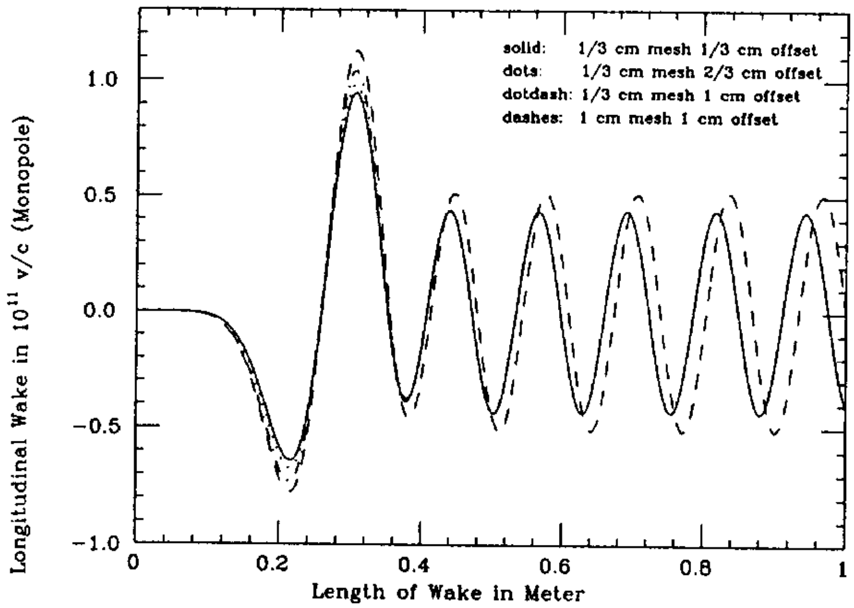

Figure 2: T3 longitudinal wake potentials for different mesh sizes and offset

potential on the offset implying that the dominant term in the longitudinal wake is the monopole term, not sensitive to the beam displacement. Also a decrease in the mesh size has little effect on both the peak values and frequency of oscillations. Fig. 3 is a pure longitudinal monopole obtained from MAFIA with $a = 0$ for two different mesh sizes. This plot should be compared with the longitudinal monopole wake potential as obtained by TBCI and shown in Fig. 4. Here the peaks for T3 vary from -0.644 to 0.980 ($\times 10^{11}$) v/c, and the peaks for TBCI vary from -0.619 to 0.922 ($\times 10^{11}$) v/c. The two results are in good agreement. The solid curve in Fig. 3 is the closest to the TBCI result both in peaks and frequency of oscillations. This curve corresponds to a smaller mesh size. A further decrease in mesh size in T3 had no effect on these results.

The dipole components for T3 and TBCI are shown in Figs. 5 and 6 respectively. The T3 dipole component is obtained by subtracting the longitudinal wake at no offset from the longitudinal wake with a beam offset. Such a subtraction can lead to some noise error as observed in the tail of Fig. 5.

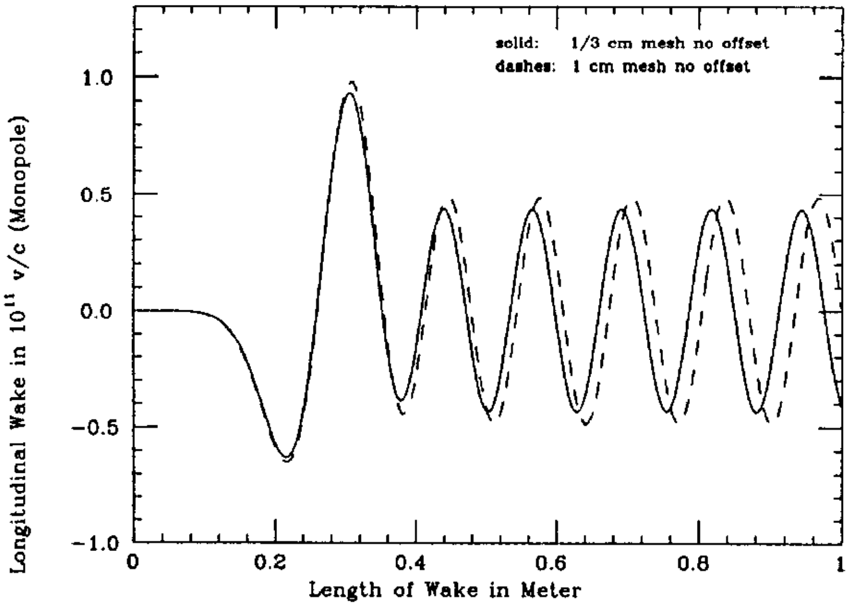

Figure 3: T3 longitudinal monopole wake potential with zero offset

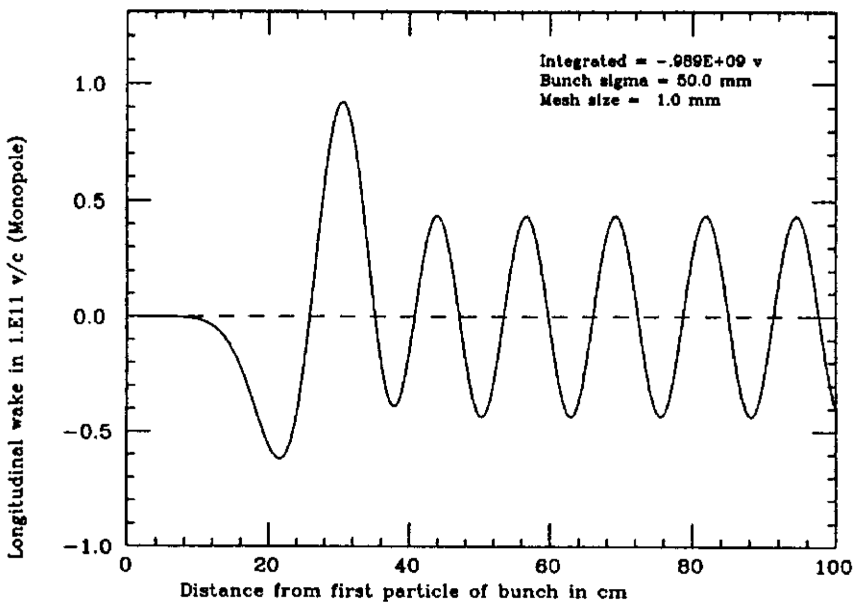

Figure 4: TBCI longitudinal monopole wake potential

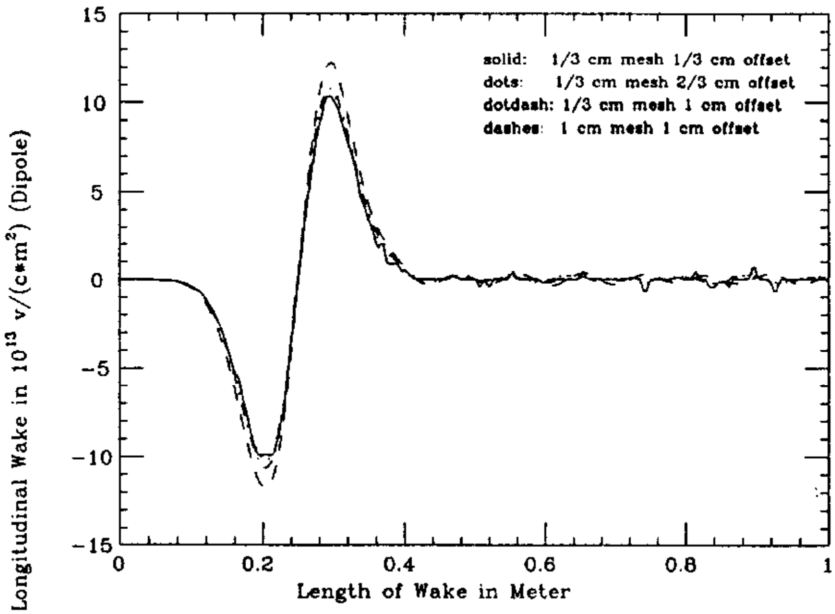

Figure 5: T3 longitudinal dipole wake potentials for different offsets and mesh sizes

From Eq. (3) the longitudinal dipole wake potential is obtained from:

$$W_1'(s) \approx \frac{F_z(s;a) - F_z(s;0)}{eqa^2} \ . \tag{6}$$

The subtraction noise error is of the order a^2. For this reason the result of the subtraction error is worse for $a = 1/3$ cm. Again the amplitudes obtained using TBCI, from -9.535 to 9.790 ($\times 10^{13} v/(cm^2)$), compare very closely to the solid curve results obtained from MAFIA. In calculating the dipole component of the longitudinal wake we have assumed that the contribution from higher order terms is negligible, something that was verified in Fig. 2.

The dipole component of the longitudinal wake can be calculated in a more straightforward manner. This is possible by considering the solution of the longitudinal wake at the beam location with half the cavity.

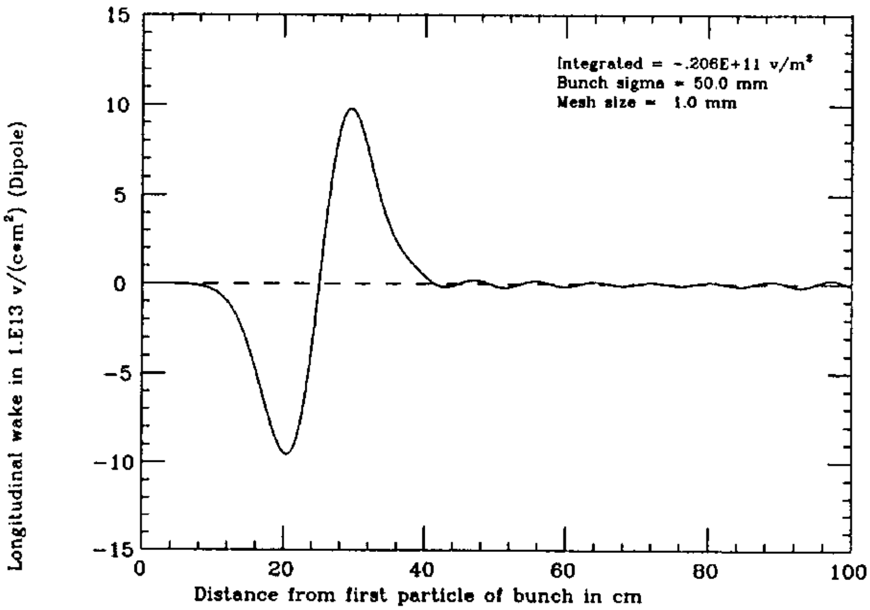

Figure 6: TBCI longitudinal dipole wake potential

2.2 Transverse Wakes

Since T3 does not compute each term separately, we need to first verify whether the dominant term in the summation of Eq. (2) is the dipole term. Figure 7 shows the transverse wake for different mesh sizes and offsets as obtained by MAFIA. The transverse wake potential is computed from the transverse wake force by dividing by a. The $W_1(z)$ peak decreases significantly as the mesh size and the offset decrease. However, a further decrease in mesh size has little effect on the result implying the presence of a dominant dipole term. The TBCI result is shown in Fig. 8. Both TBCI and MAFIA predicts almost the same result, 0.744 and 0.786 respectively. The oscillations in the tail of the wake disappear at one point in T3 raising some doubts on their validity.

3 The Asymmetric Cavity

This is a reproduction of the example given in the MAFIA manual [4]. The problem geometry is shown in Fig. 9. Because of the lack of cylindrical symmetry, this model can only be run with T3.

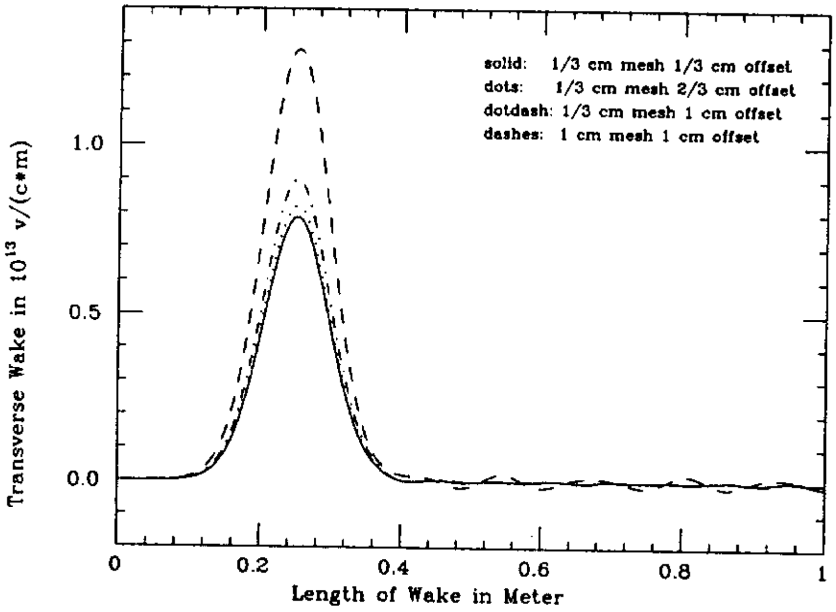

Figure 7: T3 transverse dipole wake potential for different offsets and mesh sizes

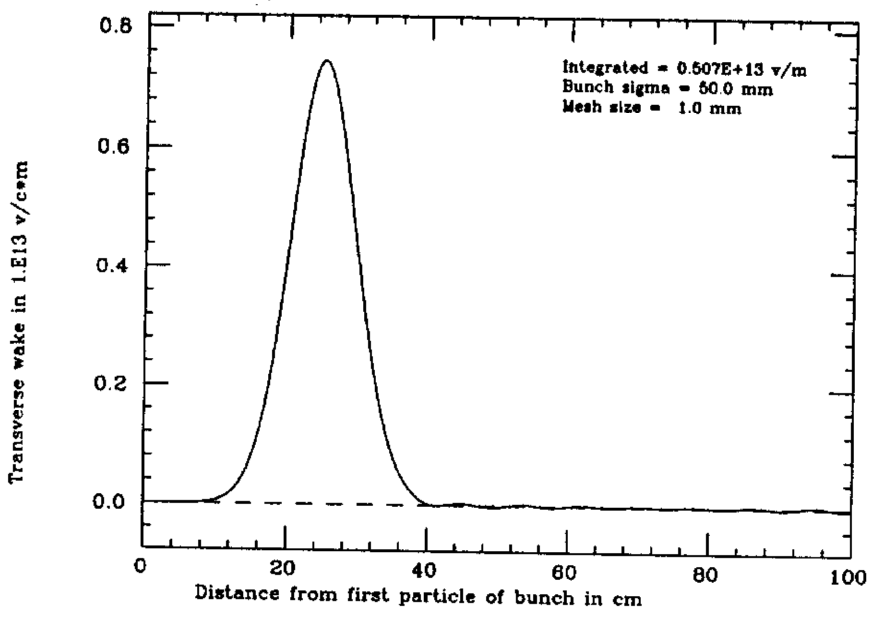

Figure 8: TBCI transverse dipole wake potential

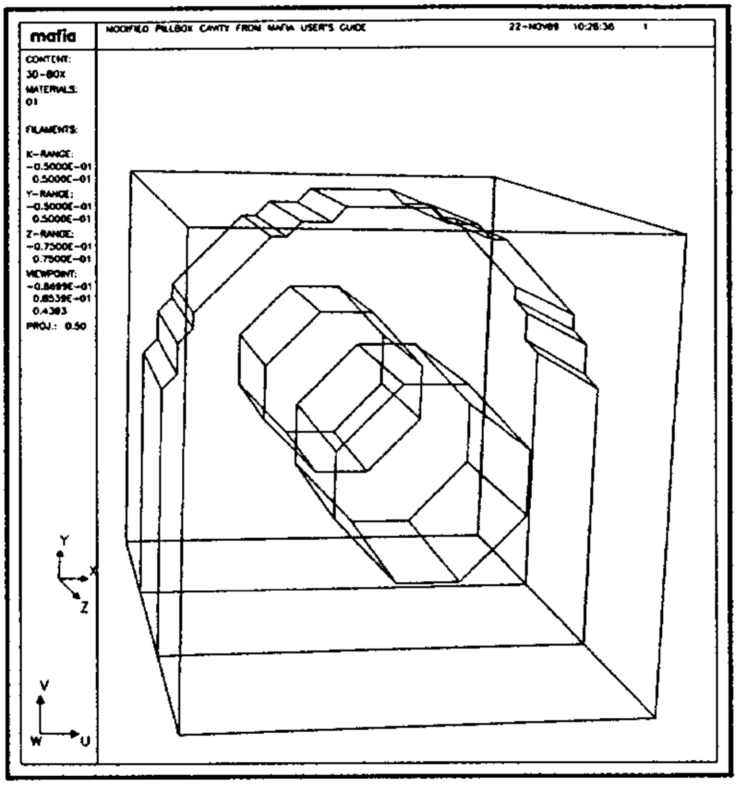

Figure 9: Asymmetric Cavity

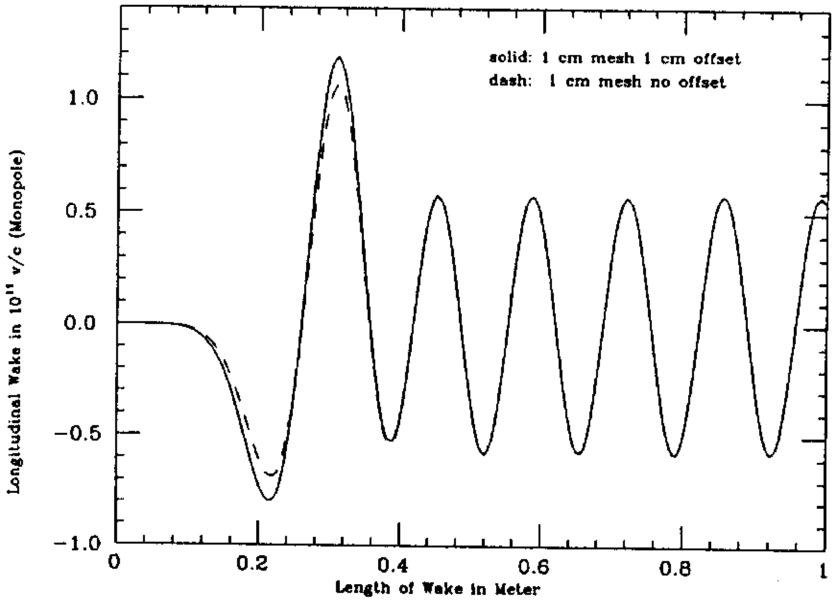

Figure 10: T3 longitudinal wake potential with and without displacement

3.1 Longitudinal Wakes

Figure 10 shows the longitudinal potential with and without beam displacement. One sees negligible difference in two cases. This is expected because the contribution is mainly the monopole contribution. The peak in amplitude corresponds to what is given in the MAFIA manual [4].

The longitudinal dipole wake is given in Fig. 11 for two mesh sizes and offsets. To see why the solid curve is not smooth we refer to the plot shown in Fig. 7. Again the result of subtraction with $a = 1/3$ cm leads to more noise error than the one with $a = 1$ cm, for the reason mentioned in section 2.1.

3.2 Transverse Wakes

The transverse wake, mainly dipole, is illustrated in Fig. 12 for two mesh sizes and offsets. Similar to previous results, the transverse wake decreases and converges to some value as the mesh size and offset decrease.

The wakes for this asymmetric cavity are in fact not much different from

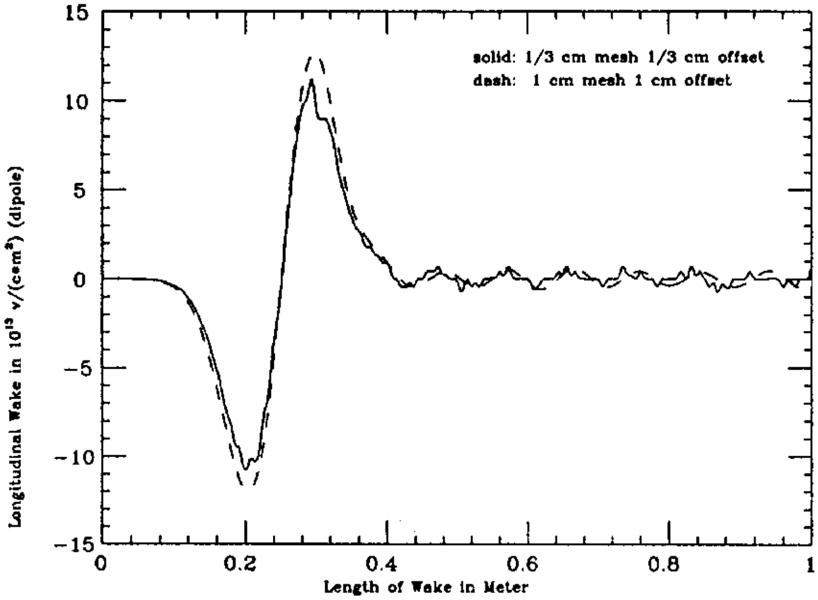

Figure 11: T3 longitudinal dipole wake potential for two different mesh sizes and offsets

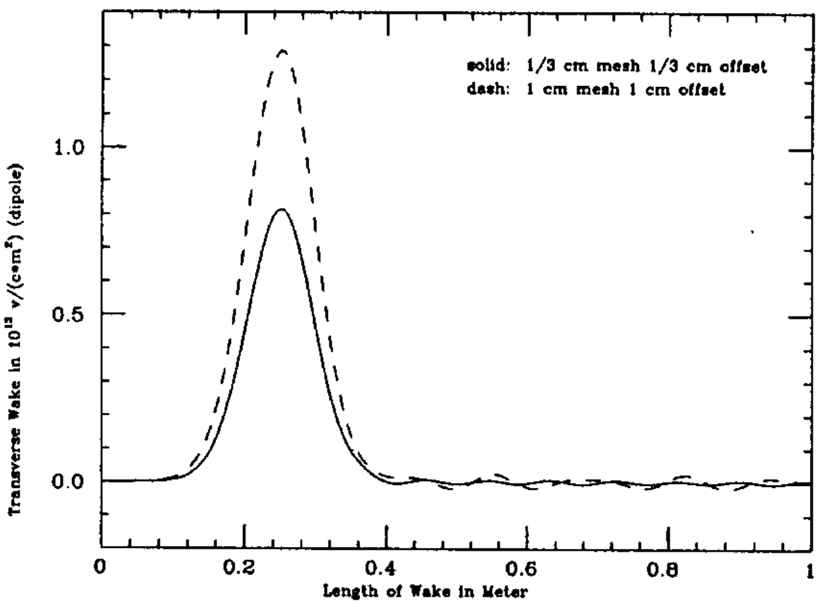

Figure 12: T3 transverse dipole wake potential

those of the symmetric pillbox. For this reason, in many cases it is quite good to approximate the asymmetric discontinuity by a symmetric continuity and use TBCI instead.

4 Summary and Conclusion

This is our first comparison of MAFIA T3 results against TBCI. Our results show a very good agreement both in peak amplitudes of the wake potential and the general shape of the wake functions. The oscillations in the tail of the functions obtained from T3 are not reliable. The spectrum of the wake, necessary for impedance calculations, depends very much on how sufficiently long the wake is to avoid truncation errors. This type of impedance calculations may not be very accurate in time domain and a better solution would be a 3-D eigen-modes solver like MAFIA-E31. We also conclude from these results that, although a small offset will reduce the contribution of higher multipoles, this will increase the computation error in subtractions and divisions. One should therefore be aware of the trade offs in selecting the amount of offset.

The calculations in this paper were done using the TBCI and MAFIA codes available at Fermi.

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